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ORIGINAL ARTICLE

A study of school mathematics curriculum enacted by competent teachers in Singapore secondary schools

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Abstract A study of school mathematics curriculum enacted by competent teachers in Singapore secondary schools is a programmatic research project at the National Institute of Education (NIE) funded by the Ministry of Education (MOE) in Singapore through the Office of Education Research (OER) at NIE. The main goal of the project is to collect a set of data that would be used by two studies to research the enacted secondary school mathematics curriculum. The project aims to examine how competent experienced secondary school teachers implement the designated curriculum prescribed by the MOE in the 2013 revision of curriculum. It does this firstly by examining the video recordings of the classroom instruction and interactions between secondary school mathematics teachers and their students, as it is these interactions that fundamentally determine the *nature* of the actual mathematics learning and teaching that take place in the classroom. It also examines content through the instructional materials used—their preparation, use in classroom and as homework. The project comprises a video segment and a survey segment. Approximately 630 secondary mathematics teachers and 600 students are participating in the project. The data collection for the video segment of the project is guided by the renowned complementary accounts methodology while the survey segment adopts a self-report questionnaire approach. The findings of the project will serve several purposes. They will provide timely feedback to mathematics specialists in the MOE, inform pre-service and professional development programmes for mathematics teachers at the NIE and contribute towards articulation of "Mathematics pedagogy in Singapore secondary schools" that is evidence based.

Keywords Mathematics curriculum · Competent and experienced mathematics teachers · Perceptions of students about mathematics teaching · Teacher-intended curriculum · Secondary schools · Singapore

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Introduction

The main goal of this programmatic research project is to examine how experienced secondary school teachers implement the designated curriculum prescribed by the Ministry of Education in the 2013 revision of curriculum. This research is timely as it will be carried out in 2016–2018, 3 to 4 years after the revised curricula for mathematics has been introduced. The findings will be pertinent for subsequent revision of the curricula.

Shaped by the research interests of a group of colleagues in the Mathematics and Mathematics Education (MME) Academic Group at the National Institute of Education (NIE) in Singapore, the project belongs to the CORE Research Programme of the Office of Education Research (OER) at NIE. It is a special focus project of system studies in pedagogical and educational outcomes. It focuses on understanding what goes on and what works in Singapore's classrooms—more specifically, the instructional core (City et al. 2009). The instructional core comprises

> "the teacher and the student in the presence of content … it is the relationship between the teacher, the student, and the content – not the qualities of any one of them by themselves – that determines the nature of instructional practice, [even though] each … has its own particular role and resources to bring to the instructional process" (City et al. 2009, pp. 22-23).

The project is about the interactions between secondary school mathematics teachers and their students, as it is these interactions that fundamentally determine the *nature* of the actual mathematics learning and teaching that take place in the classroom. It also examines the content through the instructional materials used—their preparation, use in classroom and as homework. Such studies are crucial for the Ministry of Education (MOE) in Singapore and schools to gain a better understanding of what works in the instructional core in their classrooms and schools. This is critical for the development of their education system.

Review of literature

In this section, the review of literature is in three parts. We first outline the findings of the CORE 2 research conducted by David Hogan and colleagues at NIE from 2006 to 2012 concerning mathematics lessons in Singapore secondary classrooms. Next, we review a model of curriculum enactment and introduce the concept of "teacher-directed' curriculum that guides the research in the project. Lastly, we review selected literature on teaching of mathematics that foregrounds our concept of pedagogy in the enacted curriculum.

What do the findings of CORE 2 tell us about mathematics teaching and learning in Singapore secondary mathematics classrooms?

As part of the CORE 2 research led by David Hogan, the quality of the enacted curriculum in Secondary 3 (grade 9) mathematics lessons in Singapore was assessed using criteria and standards identified by Hattie in Visible Learning (2012). More than 1000 Secondary 3 students in 30 schools drawn from a representative random stratified sample of Secondary 3 schools and 31 mathematics teachers from the Express and Normal (Academic) courses of the study were involved in the study. Data was gathered from student surveys, video records of lessons, and post-lesson teacher interviews.

The findings of the research specific to secondary three mathematics lessons were as follows:

1. Teachers focused more on procedural knowledge than conceptual knowledge and only engaged students in domain-specific knowledge practice in about a third of the instructional time of a typical lesson. Of the domain-specific knowledge practices, knowledge representation was emphasized. They also found that epistemic talk—systematic talk about knowledge that is critical to visible teaching and learning and to enhancing student understanding and skill formation—was lacking in the lessons. There was also lack of formative monitoring that could make student learning visible. Instead, procedural learning support was evident as teachers often helped with the "how to do" steps (Hogan et al. 2013a).
2. Students were engaged in doing performative tasks (77.3%) more often than knowledge building tasks (22.7%) (Hogan et al. 2013b). A performative task mainly entails the use of lower order thinking skills such as recall, comprehension, and application of knowledge while a knowledge building task calls for higher order thinking skills such as synthesis, evaluation, and creation of knowledge.
3. There was a dominant performative orientation of pedagogical practice in Singapore (Hogan et al. 2013c, p. 100), and this may explain Singapore's stellar performance in international studies.

While the findings of the CORE 2 research provided some insights about the widespread orientation of our secondary school mathematics classroom teaching and learning, they do not inform us about what our competent experienced teachers do when compared to the broad base of teachers studied in CORE 2. It is also not possible to infer how the "performative orientation" has contributed to our students' performance in PISA studies. Do our competent experienced teachers engage students in meta-cognition an essential element of twenty-first century competencies—Civic Literacy, Global Awareness and Cross-Cultural Skills; Critical and Inventive Thinking; Communication, Collaboration and Information Skills—as envisioned by the Singapore Ministry of Education (MOE nd)? How does the prescribed curriculum of the Ministry of Education for mathematics translate into teacher plans and classroom actions of competent experienced teachers? The current project builds on the findings of CORE 2, to study the pedagogies adopted by competent experienced secondary mathematics teachers when enacting the curriculum. The findings will provide mathematics educators, curriculum developers and policy makers with much valued insights about "the best that takes place in our secondary mathematics classrooms" from the perspectives of both teachers and their students.

Curriculum enactment process

In the context of this project, teacher-intended curriculum represents plans of the teacher about what to teach and how he/she plans to teach it; teacher-enacted curriculum represents what is taught during the lesson, and designated curriculum is the prescribed (official) curriculum by the MOE, in terms of syllabuses and guidelines. In our conceptualisation of the curriculum enactment process, we draw upon the visual model created by Remillard and Heck (2014) shown in Fig. 1. Kaur (2014) in her review of research on the enactment of school mathematics curriculum in Singapore noted that the model shown in Fig. 1 was rigorous for use in researching the curriculum enactment process in Singapore as it linked the official and operational curriculum in mathematics classrooms.

The model shows that as teachers draw on the designated curriculum (which in the case of the project is the Mathematics Syllabus for Secondary Schools (Ministry of Education 2012)) along with other resources (particularly instructional materials) to design instruction, they create what we would refer to as “teacher-intended” curriculum in the context of the project. It includes the interpretation and decisions teachers make to envision and plan instruction. Remillard and Heck (2014) noted that this form of curriculum is difficult to document as part of it exists in the most detailed form in the teacher’s mind. Nevertheless, detailed teacher plans and post-lesson video stimulated interviews with the teachers may offer an opportunity to capture the teacher-intended curriculum and its enactment succinctly.

Despite its importance, the enacted curriculum is multi-faceted and difficult to measure and study. The number of potential features may be numerous and at times even difficult to define let alone measure. Nevertheless, some prominent dimensions that have been studied centre around the mathematics, the pedagogical moves and the use of resources and tools (Remillard and Heck 2014). The following elaborates each of the above dimensions further.

1. The mathematics—this refers to the content and nature of the mathematics topics and practices that are emphasized and valued. For example, Hiebert et al. (2003)

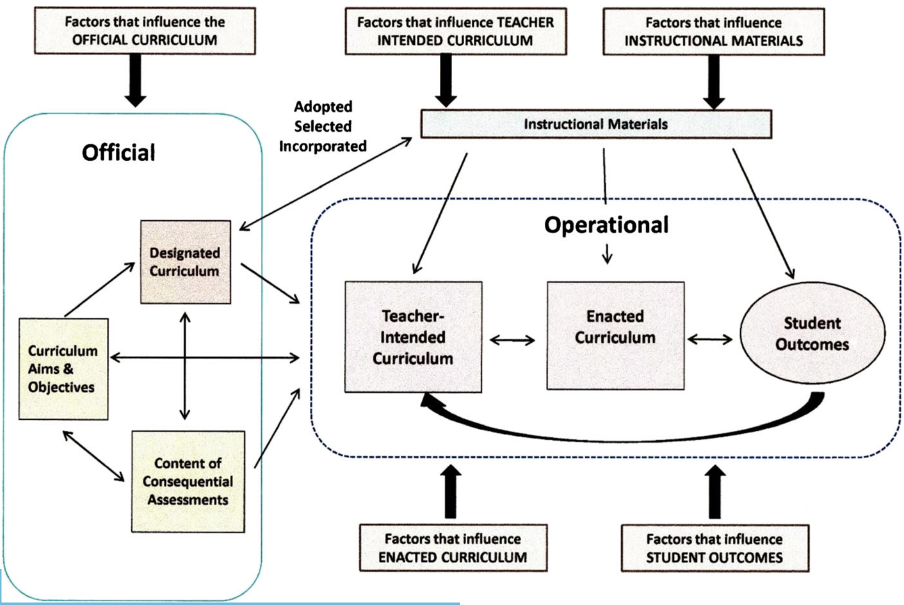

Fig. 1 Model of the curriculum enactment process (Remillard and Heck 2014, p. 709)

and Stigler et al. (1999) study of how mathematics content was presented to students considered features like demonstration, practice, recall of concepts, conceptual connections and proof. Boaler and Staples (2008), Eisenmann and Even (2009) and Stein et al. (1996) added an addition focus on the level of cognitive expectations.

2. The teacher's pedagogical moves—refer to teacher's actions, both intentional and unintentional, which shape what mathematics is addressed, including how it is represented and investigated. Teacher moves also influence how classroom interactions are structured, the kinds of interactions that are valued and which tools and resources are used during instruction. In a review of research on the teacher's role in mathematics discourse, Walshaw and Anthony (2008) identify three distinct roles that teachers play to shape mathematics classroom discourse: (i) identifying and drawing out specific mathematical ideas, (ii) fine-tuning the mathematical language and conventions used and (iii) shaping mathematical argumentation as it develops. For this project, we build on earlier studies of instructional cycles (Seah et al. 2006) and content learning discourse (Kaur 2013) done in Singapore. The instructional cycles comprised combinations of segments such as [D]—whole class demonstration, [S]—seatwork, [R]—whole class review of student work. The content learning discourse was dominated by teacher talk and student listening.
3. The use of resources and tools—refer to physical, technological, linguistic and cognitive tools that might be used during instruction by both teacher and students. Tools include instructional resources, like textbooks, as well as concrete resources like calculators, computers and manipulatives such as alge-cards and algebra-tiles. In Singapore, tools are often introduced into the classroom through teachers' moves and influence how the mathematics is represented and forms of student engagement, as well as the nature of the classroom interactions. For example, Leong et al. (2015) show how alge-cards helped students factorize quadratic expressions meaningfully and Kaur et al. (2006) studied the role of textbook in two grade eight mathematics classrooms.

Perspectives of mathematics teaching

Teaching is a cultural activity (Stigler and Hiebert 1999), and there are varying eastern and western perspectives about mathematics teaching. Two significant dichotomies that exist between the perspectives of the west and east are (i) the product versus process dichotomy and (ii) the rote learning versus meaningful learning dichotomy (Leung 2001). Anthony and Walshaw (2009) recognized that classroom teaching is a complex activity and that the classroom learning community is neither static nor linear. Based on their research on the western perspective of mathematics teaching, they offer ten principles of effective pedagogy, amongst which are (i) arranging for learning—mathematics learning experiences; (ii) mathematical communication with a focus on mathematical argumentation; (iii) mathematical tasks that influence how students come to view, develop, use and make sense of mathematics and (iv) tools and representations that support students' thinking.

The three decades of research by Schoenfeld (2011) in the USA on mathematical problem solving and mathematics instruction affirms that moment-to-moment decision making in teaching can be modelled as a function of teachers' resources (especially knowledge), orientations (especially beliefs) and goals. He advocates that the five dimensions of mathematically powerful classrooms are (i) the mathematics context; (ii) cognitive demand; (iii) access to mathematical content; (iv) agency, authority and identity and (v) uses of assessment.

Kaur (2009) in her study of grade eight mathematics lessons in the east (Singapore), in which she juxtaposed student and teacher perceptions about effective lessons, found that these lessons had the following characteristics: (i) whole-class demonstration (exposition) where the teacher explained clearly the concepts and steps of procedures, made complex knowledge easily assimilated through demonstrations, use of manipulatives, real-life examples and introduced new knowledge; (ii) seatwork and out of class assignments where the teacher gave clear instructions related to mathematical activities for in class and after class work, provided interesting activities for students to work on individually or in small groups and provided sufficient practice tasks for preparation towards examinations and (iii) review and feedback where the teacher reviewed past knowledge and used student work or group presentations to give feedback to individuals or the whole class.

From the findings of Kaur (2009), it is apparent that there is emphasis on the development of skills, in Singapore classrooms, but to say that understanding is not emphasized may not be valid. Though algorithms lead to proficiency of skills, they can also contribute to understanding, as exemplified by Fan and Bokhove (2014). Fan and Bokhove have aptly illustrated how algorithms are powerful in-roads for conceptual understanding with their three-level model of learning:

1. Cognitive level 1: knowledge and skills—mainly involves direct teaching and teachers may tell, demonstrate and engage students in drill and practice and or remediation to correct their mistakes.
2. Cognitive level 2: understanding and comprehension—involves explaining where teachers explain the why of the steps in the algorithm and perhaps even why it works, involves justifying where teachers engage students to make sense of how the algorithm was derived logically or even prove it and involves making connections where teachers help students connect the algorithm with their past knowledge.
3. Cognitive level 3: evaluation and construction—involves guided exploration where teachers create learning activities for students to explore and obtain the algorithm, followed by open exploration where teachers create learning activities for students to explore and obtain the algorithm.

Summary

From the findings of CORE 2, we know there is a dominant performative orientation of pedagogical practice involving student classroom activities and discourse in Singapore. However, from these findings, we are unable to infer if the performative orientation also pervades the classrooms of experienced secondary mathematics teachers in Singapore. Moreover, the dominant use of performative mathematical tasks and performative orientation of classroom pedagogy alone cannot explain the success of Singapore students in PISA. There is a need to examine how experienced secondary mathematics teachers in Singapore enact the school mathematics curriculum, so that we have knowledge about the upper bound of pedagogies adopted by secondary mathematics teachers in Singapore schools, thereby illuminating the potential of the prescribed curriculum by the Ministry of Education.

For the last 25 years, the framework for school mathematics curriculum has placed emphasis on five factors: concepts, skills, processes, meta-cognition and attitudes (Ministry of Education 2012) that contribute towards the primary goal of teaching mathematics in Singapore schools which is mathematical problem solving. How has this emphasis shaped the practice of our mathematics teachers? If we map the pedagogy of our mathematics teachers against the five dimensions of mathematically powerful classrooms advocated by Schoenfeld (2011), what are the outcomes? If we look deeper at why Singapore teachers engage their students in working with algorithms or homework or how they use their mathematics textbooks for learning, what can we infer about how mathematics may be learnt? All of the above thoughts have shaped two studies for the programmatic research project:

Study 1: Pedagogies adopted by experienced mathematics teachers when enacting the curriculum

Study 2: Experienced secondary school mathematics teachers' use of instructional materials for the enactment of the curriculum

Research design

Research questions

Study 1: Pedagogies adopted by experienced mathematics teachers when enacting the curriculum

The six research questions guiding study 1 are:

How do experienced mathematics teachers introduce concepts to students or engage students in constructing concepts in the Express/Normal (Academic) course/Normal (Technical) course of study? What resources or tools do they use? What are the factors that influence their respective approaches? [This question partially answers the concern: how learning experiences introduced in the 2012 syllabuses are being integrated into their mathematics lessons?]

How do experienced mathematics teachers engage students in developing fluency with skills in computing or manipulating mathematical tasks in the Express/Normal (Academic) course/Normal (Technical) course of study? What resources or tools do they use? What are the factors that influence their respective approaches?

What are the mathematical processes commonly emphasized by experienced mathematics teachers in the Express/Normal (Academic) course/Normal (Technical) course of study? What are the factors that influence their respective approaches?

How do experienced mathematics teachers facilitate the development of metacognitive strategies amongst their students in the Express/Normal (Academic) course/Normal (Technical) course of study? What are the factors that influence their respective approaches? [This question partially answers the concern: how are these teachers developing twenty-first century skills during their mathematics lessons?]
How do experienced mathematics teachers imbue desired attitudes for the learning of mathematics amongst students in the Express/Normal (Academic) course/ Normal (Technical) course of study? What are the factors that influence their respective approaches?
What are the perceptions of students in the Express/Normal (Academic) course/ Normal (Technical) course of study about good mathematics lessons?

Study 2: Experienced secondary school mathematics teachers' use of instructional materials for the enactment of the curriculum

The three research questions guiding study 2 are as follows:

How do experienced teachers select instructional materials for use in their lessons preparation and/or classroom work?
How do the experienced teachers modify the selected instructional materials? What are their guiding principles as they do so?
What are the characteristics of "instructional materials" that will fulfil the twin objectives of (i) helping experienced teachers enact worthy instructional goals of teaching mathematics and (ii) helping students achieve desirable outcomes?

Research methodology

Both studies draw on the same dataset and therefore are being carried out concurrently.

Data collection design

The project comprises two parts, a video segment and a survey segment, where the survey segment is dependent on the findings of the video segment. The video segment documents the pedagogy of experienced secondary mathematics teachers while the survey segment aids in establishing how uniform the pedagogy of experienced teachers is in the mathematics classrooms of Singapore schools. The video segment of the study is adopting the complementary accounts methodology developed by Clarke (1998, 2001), a methodology which is widely used in the study of classrooms across many countries in the world as part of the learner's perspective study (Clarke et al. 2006). This methodology recognizes that only by seeing classroom situations from the perspectives of all participants (teachers and students) can we come to an understanding of the motivations and meanings that underlie their participation. It also facilitates practice-oriented analysis of learning. For the survey, the project is adopting a self-report questionnaire to collect data on teachers' enactment of their "teacher-intended" curriculum.

Participants

Overall, approximately 630 secondary school mathematics teachers and 600 secondary school students in Singapore are participating in the project. Thirty competent experienced teachers (10 Express course of study, 4 Integrated Programme, 8 Normal (Academic) course of study and 8 Normal (Technical) course of study) and approximately 600 (in each class about 20 students, who volunteer to be the focus students will be interviewed) students in their classrooms are participating in the video segment of the project. In the context of the project, a competent experienced teacher is one who has taught the same course of study for a minimum of 5 years and is recognized by the school/cluster as a competent teacher who has developed an effective approach of teaching mathematics. These teachers were nominated by their respective school leaders, and the research team followed up on the nominations and interviewed the teachers. A strict requirement for participation in the study was that the teacher had to teach the way she/he did all the time, that is, no special preparation was expected.

For the survey segment of the project, 600 secondary school mathematics teachers, purposefully sampled and representative of the profile of mathematics teachers in Singapore secondary schools, will participate in the project. The survey segment of the project follows from the video segment of the same, as the findings from the video segment will shape the content of the survey.

Data collection procedure

For the video segment of the project, data is generated as follows:

1. A three-camera (teacher camera, student camera, whole-class camera) approach is adopted. The teacher camera maintains the teacher in centre screen as large as possible and captures all the gestures, tools and equipment the teacher uses in the lesson. The student camera keeps in view two students who are sitting adjacent to each other and focuses on their actions during the lesson. The whole-class camera captures the corporate behaviour of the class and is set at the front looking at the class such that it represents the "teacher's-eye view" of the class.
2. Sequences of lessons (for a complete topic) taught by the participating teachers are recorded.
3. Post-lesson interviews are held for the pair of focus students of the lesson. The purpose of the interviews is twofold: the first is to check if the student can complete correctly mathematical tasks that draw on the knowledge that was developed during the lesson, and the second is to document the effectiveness of the lesson from the perspective of the student. The stimuli for the interviews are the video recordings from the teacher and student cameras for the lesson. During the interviews, students are prompted by the following prompts, which are adapted from Clarke et al. (2006):

   Please tell me what you think that lesson was about (lesson content/lesson purpose).
   How, do you think, you best learn something like that?

What were your personal goals for that lesson? What did you hope to achieve? Do you have similar goals for every lesson?
Here is the computer. You can play your math lesson we recorded today. I want you to play the recording and stop it at sections that are important to you. Tell me why these sections are important to you.
After watching the recorded lesson, is there anything you would like to add to your description of what the lesson was about?
What did you learn during that lesson?
Would you describe that lesson as a good one for you? What has to happen for you to feel that a lesson was a "good" lesson? Did you achieve your goals?
What are the important things you should learn in a mathematics lesson?
Was this lesson a typical [geometry, algebra, etc.] lesson? What was not typical about it?
How would you generally assess your own achievement in mathematics?
Do you enjoy mathematics and mathematics classes?
Why do you think you are good [or not so good] at mathematics?
Do you do very much mathematical work at home? Have you ever had private tutoring in mathematics or attended additional mathematics classes outside normal school hours?

4. Post-lesson interviews are also held for the teacher periodically (about 4 times during their participation). The stimuli for the interviews are video recordings from the teacher and whole-class cameras. During the interview, the teacher is prompted by the following prompts which are adapted from Clarke et al. (2006):

   Please choose a lesson from those you have taught on the topic that you'd like to talk with me about. What were your goals for this lesson? You may include both content and non-content goals.
   Did you use all the materials that had you intended to use for the lesson?
   Do you think you have achieved your goals that you have set out to achieve? How were the goals achieved?
   What is the most "ambitious"/challenging thing you did in the lesson earlier? How do you think it went?
   Do you think your students have achieved these goals?
   Can you share with me what the highs and lows of this lesson were?
   Here is the computer. You can play your math lesson we recorded today. I want you to play the recording and stop it at sections that you think illustrate how you achieved the goals you have shared with me just now.
   Is there a part of the lesson that you like best? Please show me the video segment of it. Can you explain a little more why you like this part best?
   How would you rate your lesson today?
   What are some of the words you would use to describe your lesson today?

5. Student written work—all the written work done by the focus students during and following the lesson are digitized and labelled with pseudo-names.
6. Teacher goals—for every lesson conducted by the teacher, the lesson objectives and goals of the lesson are collected from the teacher using a simple survey form.

7. Detailed field notes are recorded by the research assistants present during the lesson.

For the survey segment of the project, data is generated using a self-report questionnaire. The design of the questionnaire is guided by (i) the prominent dimensions of the enacted curriculum that are researchable vis-a-vis: the mathematics, teacher's pedagogical moves and the use of resources and tools, which are researchable (Remillard and Heck 2014) and (ii) the framework for school mathematics in Singapore (Ministry of Education 2012). Findings from the respective video segments of the study will contribute towards the content of the questionnaire items.

Data analysis

The data from the video segment will be coded using both inductive and deductive approaches. We draw on two distinct sources to inform our derivation of codes. The first is the five dimensions of mathematically powerful classrooms by Schoenfeld (2011). An overview of the five dimensions is shown in Table 1. The second is an integration of the features of enactment that are researchable (Remillard and Heck 2014) and the Singapore school mathematics framework (Ministry of Education 2012) which is shown in Table 2.

We are using an activity segment as an appropriate unit of analysis. According to Stodolsky (1988),

> In essence, an activity segment is a part of a lesson that has a focus or concern and starts and stops. A segment has a particular instructional format, participants, materials and behavioural expectations and goals. It occupies a certain block of

Table 1 The five dimensions of mathematically powerful classrooms (Schoenfeld 2011)

The five dimensions of mathematically powerful classrooms	
The mathematics	The extent to which the mathematics discussed is focussed and coherent and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained
Cognitive demand	The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development
Access to mathematical content	The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class
Agency, authority, and identity	The extent to which students have opportunities to conjecture, explain, make mathematical arguments and build on one another's ideas, in ways that contribute to their development of agency and authority resulting in positive identities as doers of mathematics
Formative assessment	The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings

Table 2 Hybrid model for conceptualizing codes for coding of mathematics lessons

	Singapore school mathematics framework				
Researchable features of enactment	Concepts	Skills	Mathematical processes	Meta-cognition	Attitudes
Teacher's pedagogical moves	X	X	X	X	X
Use of resources and tools	X	X	X	X	

X denotes how the feature of enactment facilitates the development of an attribute of the Singapore school mathematics framework

> time in a lesson and occurs in a fixed physical setting. A segment's focus can be instructional or managerial (p. 11).

Lesson episodes will be coded for identifiable attributes such as teacher explaining a concept and students discussing attributes of a concept, etc. To establish common understanding amongst the coders, a glossary of terms will be developed accompanied by short video clips to illuminate the term. The development of the code book will be on-going as more and more lessons are coded, and the variability of teacher and student actions may also expand. The coded data will be subjected to both quantitative and qualitative analysis. Descriptive statistics will be mainly used for the quantitative analysis. It is apparent that the coding of the video-recorded data will lend itself to subjectivity. This will be minimized by establishing an acceptable interrater reliability score of 80%.

The qualitative data arising from the interviews, field notes and teaching plans of the teachers will be analysed using qualitative analysis tools. Both deductive and inductive approaches will be used to identify the categories or themes in the narrative data collected. When we have pre-determined themes and are looking for specific pieces of data, we will adopt a deductive approach and use the framework analysis (Ritchie and Spencer 1994). Otherwise, we will adopt an inductive approach and carry out content analysis (Weber 1990). In this case, the responses will be first scanned through for common themes, following which codes are generated and the data coded. Inevitably, "a progressive process of sorting and defining and defining and sorting" (Glesne 1999, p 135) will lead to the establishment of the final list of codes for the themes.

The Survey Data from the questionnaire will be both quantitative (responses according to a scale such as always, seldom, never) and qualitative (responses to open ended prompts). The quantitative data will be analysed using the software SPSS. A mixture of descriptive statistics and non-parametric inferential statistics will guide the analysis. The qualitative data will be first coded, in a manner similar to the interview data in the video segment of the study followed by similar analysis.

Research implementation schedule

The project was funded in March 2016 by the MOE in Singapore through the OER at NIE. It is the first programmatic research project at the OER in NIE. Following the ethics approval and data collection in schools approval, the research team began the video segment of the project in June 2016. By March 2017, 12 teachers and their students have participated in the video segment of the project. It is planned that by

June 2018, the video segment of the project would be completed. The survey segment of the project will follow in 2019.

Concluding remarks

This major funded project will make both national and international contributions. At the national level, the findings of the project will serve several purposes. Firstly, they will illuminate gaps between the official curriculum and the enacted curriculum. The findings will impact subsequent cycles of revision of the school mathematics curriculum. Secondly, with regard to teacher education, the findings will inform pre-service and professional development programmes for mathematics teachers at the NIE. The findings will help teacher educators use “authentic good practices” from local classrooms for critical and meaningful discourse in their programmes. Lastly, the findings will contribute towards articulation of “Mathematics pedagogy in Singapore secondary schools” that is evidence based.

At the international level, the findings will provide scholars much to deliberate on and conjecture about the success of Singapore students in international benchmark studies like the Trends in International Mathematics and Science Study (TIMSS) (Mullis et al. 2016) and Programme for International Student Assessment (PISA) (OECD 2016).

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